

LE MATRICI: L'INVERSA

$$A_4 = \begin{pmatrix} 3 & 4 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$



**sviluppando rispetto
alla prima colonna**

$$\det A_4 = +3 \cdot \begin{vmatrix} 2 & 1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 4 \end{vmatrix} = 3(-4) = -12 \neq 0$$



esiste l'inversa di A

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \cdot (\mathbf{A}^*)^T$$

$$\mathbf{A}_4 = \begin{pmatrix} 3 & 4 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{pmatrix} \quad \longrightarrow \quad \mathbf{A}^* = \begin{pmatrix} -4 & 0 & 0 & 0 \\ 8 & 0 & -12 & 0 \\ 0 & -12 & 24 & 0 \\ -3 & 9 & -12 & -3 \end{pmatrix}$$

$$(\mathbf{A}^*)^T = \begin{pmatrix} -4 & 8 & 0 & -3 \\ 0 & 0 & -12 & 9 \\ 0 & -12 & 24 & -12 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

$$(\mathbf{A}^*)^T = \begin{pmatrix} -4 & 8 & 0 & -3 \\ 0 & 0 & -12 & 9 \\ 0 & -12 & 24 & -12 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

$$\det \mathbf{A} = -12$$



$$\mathbf{A}^{-1} = -\frac{1}{12} \begin{pmatrix} -4 & 8 & 0 & -3 \\ 0 & 0 & -12 & 9 \\ 0 & -12 & 24 & -12 \\ 0 & 0 & 0 & -3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & 0 & \frac{1}{4} \\ 0 & 0 & 1 & -\frac{3}{4} \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

$$[\textcolor{red}{A} | I] \longrightarrow [I | \textcolor{red}{A}^{-1}]$$

$$\textcolor{red}{A}_{\textcolor{blue}{4}} = \begin{pmatrix} 3 & 4 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

$$[\textcolor{red}{A} | I_{\textcolor{blue}{4}}] = \left[\begin{array}{cccc|cccc} 3 & 4 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\overline{R_1} \rightarrow \overline{R_1} - 2\overline{R_2}} \begin{array}{l} \overline{R_1} \rightarrow \overline{R_1} - 2\overline{R_2} \end{array}$$

$$\left[\begin{array}{cccc|cccc} 3 & 0 & 0 & -3 & 1 & -2 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\overline{R_3} \rightarrow 2\overline{R_3} - \overline{R_2}} \begin{array}{l} \overline{R_3} \rightarrow 2\overline{R_3} - \overline{R_2} \end{array}$$

$$\left[\begin{array}{cccc|cccc} 3 & 0 & 0 & -3 & 1 & -2 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 4 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\overline{R_3} \rightarrow \overline{R_3} - \overline{R_4}}$$

$$\left[\begin{array}{cccc|cccc} 3 & 0 & 0 & -3 & 1 & -2 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\overline{R_2} \rightarrow \overline{R_2} + \overline{R_3}}$$

$$\left[\begin{array}{cccc|cccc} 3 & 0 & 0 & -3 & 1 & -2 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\overline{R_2} \rightarrow \overline{2R_2} - \overline{R_4}}$$

$$\left[\begin{array}{cccc|cccc} 3 & 0 & 0 & -3 & 1 & -2 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 4 & -3 \\ 0 & 0 & -1 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} R_1 \rightarrow \frac{1}{3}R_1 \\ R_2 \rightarrow \frac{1}{4}R_2 \\ R_3 \rightarrow -R_3 \\ R_4 \rightarrow \frac{1}{4}R_4 \end{array}} \rightarrow$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & \frac{1}{3} & -\frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{4} \end{array} \right] \xrightarrow{R_1 \rightarrow R_1 + R_4} \rightarrow$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{3} & -\frac{2}{3} & 0 & \frac{1}{4} \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{4} \end{array} \right] = \left[I_4 \mid A^{-1} \right]$$



$$A^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & 0 & \frac{1}{4} \\ 0 & 0 & 1 & -\frac{3}{4} \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{(cfr. slide 3)}$$